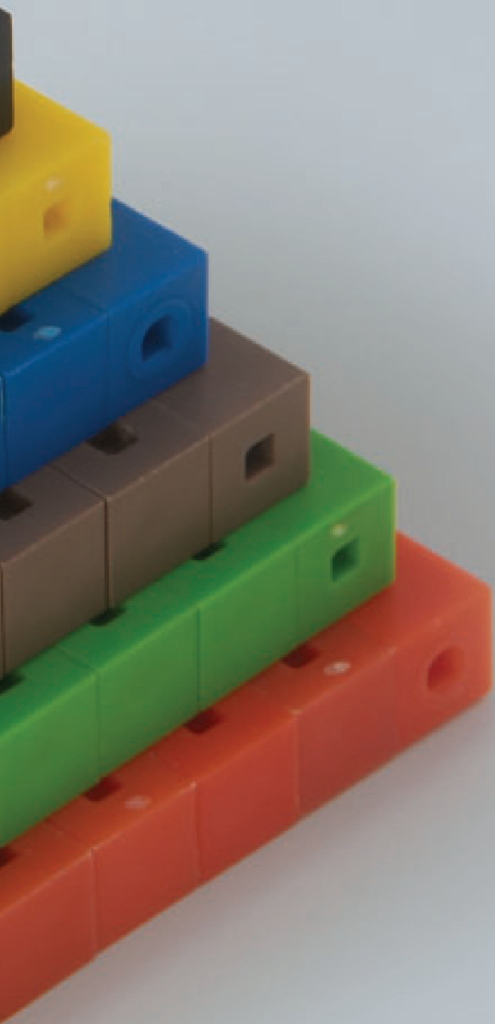


Chapter 1

Number Relationships

GOAL

You will be able to

- model perfect squares and square roots
- use a variety of strategies to recognize perfect squares
- use a variety of strategies to estimate and calculate square roots
- explain and apply the Pythagorean theorem
- solve problems by using a diagram

◄ This is a model of the pyramid at Chichen Itza, in Mexico. Each of the layers of the model is a square built from centimetre cubes. How many cubes are needed to make the model pyramid?

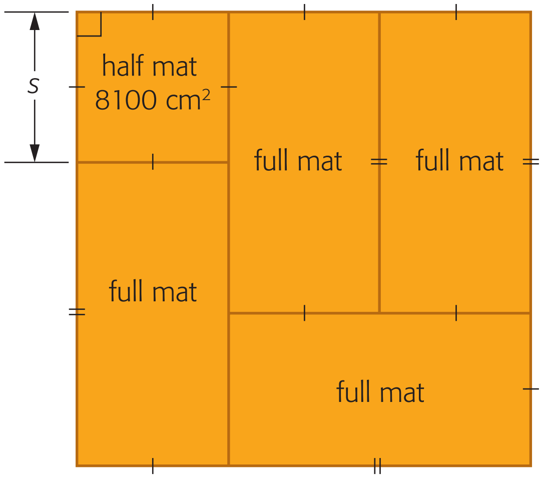
Chapter 1

Getting Started

YOU WILL NEED
- grid paper

Tatami Mats

Vanessa presented a report on Japanese tea rooms to her class. The floors are usually covered with square and rectangular tatami mats. She drew one way to cover a square floor with a square half mat and four rectangular full mats. The area of the half mat is 8100 cm^2 and is half the size of a full mat.



What are the dimensions of the mats and the room?

A. The variable *s* represents the side length of the square mat. Why can you use the equation $s \times s = 8100$ to determine the side length of the square mat?

B. How do you know that the side length of the square mat must be between 50 cm and 100 cm?

C. Is the side length of the square mat closer to 50 cm or 100 cm? Explain.

D. What is the side length of the square mat? Show your work.

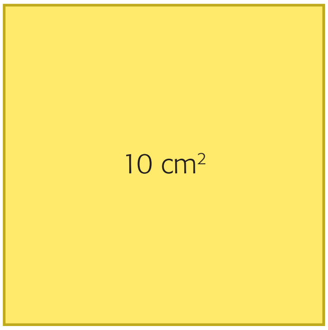
E. What are the dimensions of the rectangular mats and the room? Explain what you did.

What Do You Think?

Decide whether you agree or disagree with each statement.
Be ready to explain your decision.

1. When you multiply a number by itself, the product is always greater than the number you multiplied.

2. You can use the area to estimate the dimensions of the square.



3. This equation has no solution.
$a \times a = 12.25$

4. A right triangle has sides of 6 cm and 8 cm. The length of the third side must be about 10 cm.

1.1 Representing Square Numbers

YOU WILL NEED

- square shapes or grid paper

GOAL

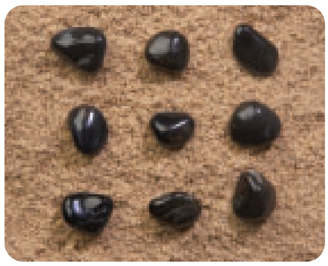
Use materials to represent triangular and square numbers.

EXPLORE the Math

Mark read that the ancient Greeks used to arrange pebbles to represent numbers. He used squares on a grid instead of pebbles to model both triangular and square numbers.

6

Six is called a triangular number because you can arrange 6 pebbles in a triangle in which each row is 1 greater than the row above it.



9

Nine is called a square number because you can arrange 9 pebbles into a 3-by-3 square.

? How can you divide a square number into two triangular numbers?

1.2 Recognizing Perfect Squares

YOU WILL NEED

- grid paper

GOAL

Use a variety of strategies to identify perfect squares.

LEARN ABOUT the Math

perfect square

the product of a whole number multiplied by itself; e.g., 49 is a perfect square because $49 = 7 \times 7$.

? Is Elena correct?

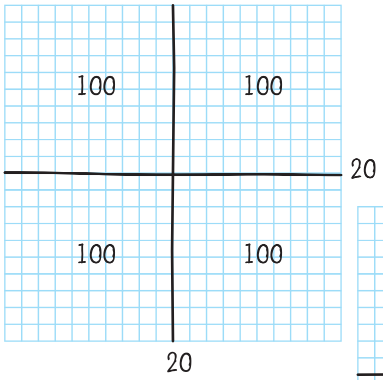
Communication | Tip

- Perfect squares can also be called square numbers.
- A 2 written above and to the right of a number shows it has been "squared."
  7^2 represents 7×7 and can be read as "7 squared."

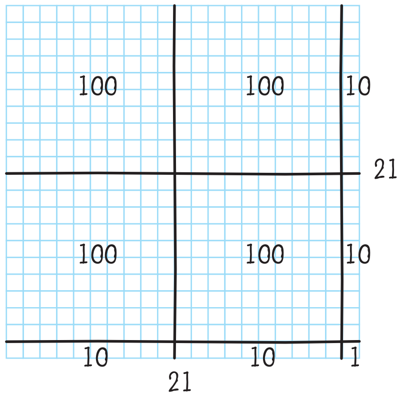
Example 1 | Identifying a perfect square using diagrams

I determined whether 441 is a perfect square by drawing a square.

Elena's Solution



Because I know $20 \times 20 = 400$, I sketched a 20-by-20 square. It has an area of 400 square units. So I know 400 is a perfect square.



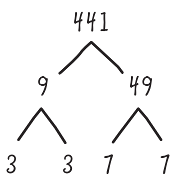
I modified my sketch to show a 21-by-21 square. $21 \times 21 = 441$, so it has an area of 441 square units.

I can draw a square with 441 square units, so 441 is a perfect square.

Example 2 | Identifying a perfect square using factors

I determined whether 441 is a perfect square using prime factors.

Mark's Solution



If 441 is a perfect square, then there are two equal factors that have 441 as a product. I decided to factor 441 to look for them.

I represented the factors in a tree diagram.
I know 441 is divisible by 9, because the sum of its digits is divisible by 9. One factor is 9. Another factor is $441 \div 9 = 49$.
9 and 49 are not equal.

I continued until all the factors were prime.

$441 = 3 \times 3 \times 7 \times 7$

I wrote 441 as the product of prime factors.

$441 = 3 \times 7 \times 3 \times 7$
$441 = (3 \times 7) \times (3 \times 7)$
$441 = 21 \times 21$

I rearranged them to create a pair of equal factors.

441 can be renamed as two equal factors, so 441 is a perfect square, and Elena is correct.

Reflecting

A. Is there a perfect square between 400 and 441? Explain.

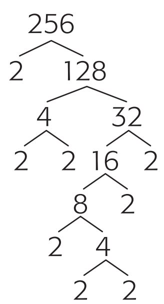
B. Would you use prime factors to determine whether 400 is a perfect square? Why or why not?

WORK WITH the Math

Example 3 | Identifying a square number using factors

Determine whether 256 is a perfect square using prime factors.

Solution



Determine the prime factors of 256 using a tree diagram. Each time you divide by a factor, you continue to get another even number. So the only prime factor is 2.

$256 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

Write 256 as the product of the prime factors.

$256 = (2 \times 2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2)$
$= 16 \times 16$
$256 = 16 \times 16$ or 16^2, so it is a square number.

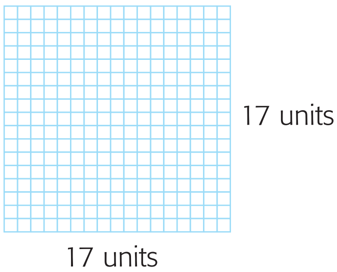
Group the factors to rename 256 as the product of two equal factors.

A Checking

1. Which numbers are perfect squares? Show your work.
   a) 64 **c)** 120 **e)** 1000
   b) 100 **d)** 900 **f)** 10 000

2. How do you know that each number is a perfect square?
   a) $1225 = 35 \times 35$
   b) $484 = 2 \times 2 \times 11 \times 11$
   c) $2025 = 45^2$

B Practising

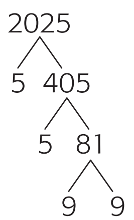


3. The area of this square is 289 square units. How do you know that 289 is a perfect square?

4. Show that each number is a perfect square.
   a) 16 **b)** 144 **c)** 1764

5. Barrett is making a display of 225 square photos of the students in his school. Each photo is the same size. Can he arrange the photos in a square? Explain.

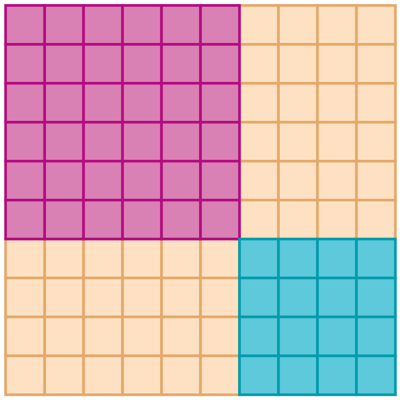
6. Calculate.
   a) 6^2 **c)** 11^2 **e)** 25^2 **g)** 100^2
   b) 9^2 **d)** 12^2 **f)** 40^2 **h)** 1000^2



7. Maddy started to draw a tree diagram to determine whether 2025 is a square number.
   How can Maddy use what she has done so far to determine that 2025 is a square number?

8. Guy says: "My street address is a square number when you read the digits forward or backward."
   Is Guy correct? Explain.

9. Star's grandmother makes square patchwork quilts. They usually contain two different squares and two congruent rectangles. What other squares and rectangles could Star's grandmother have shown in her 10-by-10 quilt?



10. **a)** How many perfect squares are between 900 and 1000? Show your work.

    b) How can you use your answers in part a) to determine the greatest perfect square less than 900 and the least perfect square greater than 1000?

11. Are 0 and 1 both square numbers? Explain.

12. When you square a number, how do you know whether the result will be odd or even?

13. How do you know that the product of two different square numbers will also be a square number? Use an example to explain.

14. Square each whole number from 11 to 20. What are the ones digits?

15. Use your answers in question 14 to predict the ones digit in each calculation. Explain what you did.

    a) 21^2 **b)** 32^2 **c)** 45^2 **d)** 58^2

16. Suppose you know the ones digit of a square number. Can you always figure out the ones digit of the number that was squared? Explain, using your answers from questions 14 and 15.

17. Because 289 has only three factors: 1, 17, and 289, explain how you can use this information to show that 289 is a square number.

1.3 Square Roots of Perfect Squares

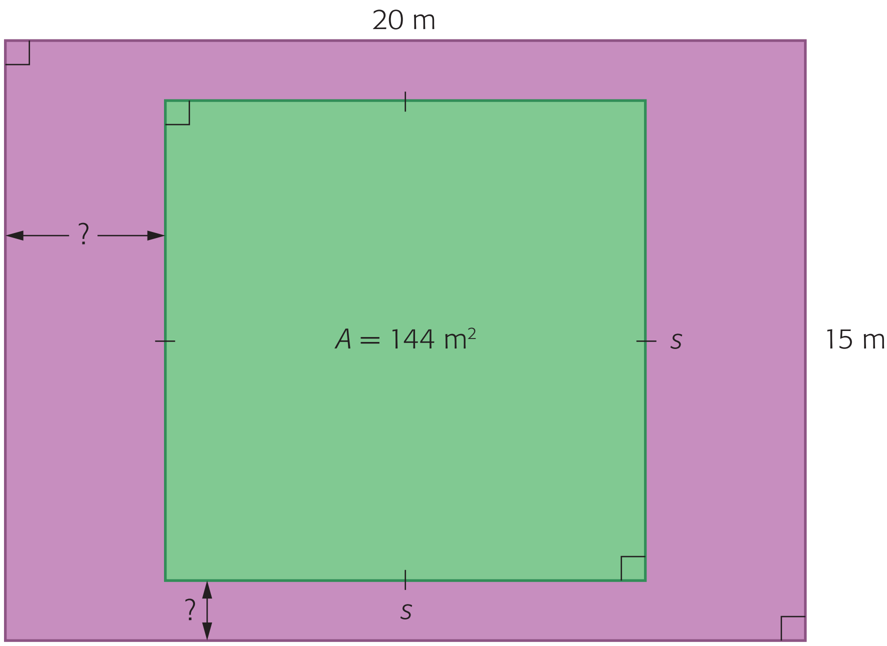
YOU WILL NEED
- grid paper

GOAL

Use a variety of strategies to determine the square root of a perfect square.

LEARN ABOUT the Math

Vanessa needs to place square mats in the middle of the gym floor. The floor is 15 m by 20 m, and the mats have an area of 144 m^2. Vanessa wants to know the distances between the sides of the floor mats and the walls of the gym. She drew a diagram to help her understand the problem.



How can Vanessa determine the distances between the sides of the floor mats and the walls of the gym?

A. How does Vanessa's diagram help her to understand the problem?

B. What does the variable s represent in Vanessa's diagram?

C. How does the equation $s \times s = 144$ help you determine the side length of the square mats?

D. Why can you solve the equation in part C by calculating the **square root** of 144? Use the diagram of the square mats to help you explain.

E. How would you solve $s \times s = 144$?

F. What is the side length of the mats?

G. What are the distances between the sides of the floor mats and the walls of the gym? Show your work.

square root

one of two equal factors of a number; for example, the square root of 81 is 9 because 9×9, or 9^2, $= 81$.

Communication | Tip

The square root symbol is $\sqrt{}$. You can write "the square root of 100" as $\sqrt{100}$.

Reflecting

H. Can you use the ones digit of 144 to predict the ones digit of the square root of 144? Explain.

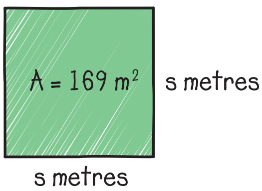
I. How can you check your answer when you calculate the square root of a number? Use $\sqrt{144}$ to explain.

WORK WITH *the Math*

Example 1 | Determining a square root by guess and test

The floor mat in rhythmic gymnastics is a square with an area of 169 m^2.
What is its side length?

Vanessa's Solution



I drew a diagram to help understand the problem.

$s \times s = 169$
$s = \sqrt{169}$

I have to determine a number that equals 169 when multiplied by itself, or squared. Each equation represents this situation.

$10^2 = 100$ too low
$20^2 = 400$ too high

The side length of the mat must be between 10 m and 20 m, but closer to 10 m than 20 m.

$3 \times 3 = 9$
$7 \times 7 = 49$

I know the ones digit of the side length must be 3 or 7, because both 3^2 and 7^2 have ones digits of 9. No other digit squared will end in 9.

Try 13.
$13^2 = 169$
So $\sqrt{169} = 13$

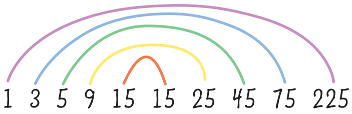
The side length of the mat is 13 m.

I tried 13 because it is between 10 and 20, but closer to 10 than 17.

Example 2 | Determining a square root by factoring

Determine the square root of 225.

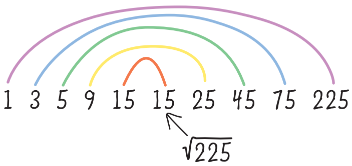
Sanjev's Solution



I made a factor rainbow to show the factors of 225.

I know 3 and 9 are factors because the sum of the digits in 225 is 9.

I know 5 is a factor because the ones digit of 225 is 5.



Because 3 and 5 are factors, 3×5, or 15, must also be a factor of 225.

The square root of 225 is 15.

The factor with an equal partner is the square root. So I can express 225 as 15×15 or 15^2.

A Checking

1. Judo mats are squares with a minimum area of 36 m^2 and a maximum area of 64 m^2. The side length of each mat is a whole number in metres.

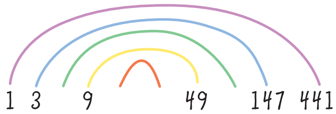
a) Sketch each possible mat on grid paper.

b) What are the possible side lengths of the mats?

2. Calculate.

a) $\sqrt{4}$ **b)** $\sqrt{16}$ **c)** $\sqrt{81}$ **d)** $\sqrt{400}$

B Practising



3. **a)** Complete the factor rainbow. Show how to use the factors to determine the square root of 441.

b) How can you check your answer in part a)?

4. Determine the square root of 729 by factoring. Show how to check your answer.

5. Maddy listed rectangles with whole number sides and an area of 64 m^2 to determine $\sqrt{64}$.

a) What other rectangles can Maddy list?

b) How can she use her complete list to determine $\sqrt{64}$?

c) Use Maddy's strategy to determine $\sqrt{144}$.

d) How is Maddy's strategy for determining a square root like Sanjev's?

6. Determine the square root of each number using mental math.

a) 1 **c)** 25 **e)** 400

b) 0 **d)** 100 **f)** 900

7. Explain how to determine each square root.

a) $\sqrt{31 \times 31}$ **b)** $\sqrt{43^2}$ **c)** $\sqrt{2 \times 2 \times 3 \times 3}$

8. **a)** The square of 32 is 1024. What is the square root of 1024?

b) The square root of a perfect square is 11. What is the perfect square?

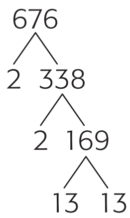
9. At the 2006 Winter Olympics in Turin, Italy, 196 Canadian athletes were at the opening ceremonies. Would they have been able to arrange themselves in a square? Explain.

10. The area of a square weightlifting platform is 16 m^2. What is the perimeter of the platform?

11. **a)** Explain how you know the square root of 225 is between 10 and 20.

b) How can you predict the ones digit of the square root of 225?

c) How can you use your answers to parts a) and b) to predict the square root of 225?



12. This tree diagram shows the prime factors of 676.

a) Is 676 a perfect square? Explain.

b) What is the square root of 676?

13. Iris said, "If the ones digit of a perfect square is 0, then the ones digit of the square root will be 0. If the ones digit of a perfect square is 1, then the ones digit of the square root will be 1 or 9."

a) Complete Iris's table.

Ones digit of perfect square	0	1	2	3	4	5	6	7	8	9
Ones digit of square root	0	1 or 9								

b) Can you always use the ones digit of a perfect square to predict its square root? Explain.

14. Determine each square root using estimation and your chart from question 13. Show your work for one answer.

a) 289 **b)** 441 **c)** 2209 **d)** 3025

15. Describe two strategies to calculate $\sqrt{324}$.

16. Determine

a) $\sqrt{100}$ **b)** $\sqrt{10\ 000}$ **c)** $\sqrt{1\ 000\ 000}$

17. Predict $\sqrt{100\ 000\ 000}$ using your answers in question 16. Explain your prediction.

18. a) Jason listed all factors of 5929.
1, 7, 11, 49, 77, 121, 539, 847, 5929
How can you determine the square root of 5929 using Jason's list of factors?

b) Show how to use squaring to check your answer.

19. A whole number has an odd number of factors. How do you know that one of the factors must be the square root?

20. Why might squaring a number and calculating the square root of a number be thought of as opposite operations? Use an example to explain.

Reading Strategy

Evaluating

Write your answer to question 20. Share it with partners. Do they agree or disagree?

1.4 Estimating Square Roots

YOU WILL NEED

- grid paper
- a calculator

GOAL

Estimate the square root of numbers that are not perfect squares.

LEARN ABOUT the Math

Kaitlyn and her father drilled a hole in the ice in the lake to measure its thickness. The ice was 30 cm thick. Their total mass is 125 kg. Can the ice support them safely? They used this formula to check.

Required thickness (cm) $\doteq 0.38\sqrt{\text{load in kilograms}}$

Communication *Tip*

- The multiplication symbol is often omitted from formulas when the meaning is clear. For example, $0.38\sqrt{\blacksquare}$ means the same as $0.38 \times \sqrt{\blacksquare}$.
- The symbol "$\doteq$" means "approximately equal to." For example, $\sqrt{2} \doteq 1.414$.

? Is the ice thick enough to support Kaitlyn and her father?

A. Draw a 10-by-10 square, an 11-by-11 square, and a 12-by-12 square on grid paper. Calculate the area of each square.

B. How can you calculate the side length of a square if you know only the area of the square?

C. Does a square with an area of 125 square units have a whole-number side length? Use your diagrams in part A to help you explain.

Calculator | *Tip*

Different calculators use different key sequences to calculate square roots.
TI-15: $\boxed{\sqrt{\ }}$ 125 $\boxed{)}\boxed{=}$
some others: 125 $\boxed{\sqrt{\ }}$

D. How can you use the side lengths of the squares you drew in part A to estimate $\sqrt{125}$?

E. Determine $\sqrt{125}$ to two decimal places using a calculator.

F. Will the ice support Kaitlyn and her father? Show your work.

Reflecting

G. Explain how to use the square key $\boxed{x^2}$ or the power key $\boxed{\wedge}$ on your calculator to check your answer in part E.

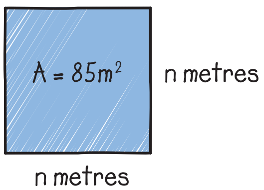
H. When you square your answer in part E, why do you not get exactly 125?

WORK WITH the Math

Example 1 | Estimating a square root using squaring

A square floor has an area of 85 m^2. About how long are its sides?

Kaitlyn's Solution



I can determine the side length of a square with an area of 85 square units by calculating $\sqrt{85}$.

$n \times n = 85$
$n^2 = 85$

The square root of 81 is 9, so the square root of 85 must be a bit more than 9.

$9^2 = 81$
$9.1^2 = 82.81$
$9.2^2 = 84.64$
$\sqrt{85} \doteq 9.2$

I squared 9.1 and 9.2.
The square of 9.2 is very close to 85.

So the square root of 85 is about 9.2.

The sides of the floor are about 9.2 m long.

Example 2 | Determining a square root using a calculator

A truck has a mass of 5000 kg. What thickness of ice is needed to support the truck? Use the formula:
Required thickness (cm) $\doteq 0.38\sqrt{5000}$.

Guy's Solution

5000 must be close to 70 because $70^2 = 4900$.

First, I estimated $\sqrt{5000}$.

Multiplying 70 by 0.38 is less than half of 70, or about 30 cm.

Then I estimated 0.38×70.

0.38 $\boxed{\times}\boxed{\sqrt{\ }}$ 5000 $\boxed{)}\boxed{=}$ 

Then I used these keystrokes and entered these numbers into my TI-15 calculator.

The ice needs to be about 27 cm thick to support the truck; 27 cm is close to my estimate of 30 cm, so the answer is reasonable.

Calculator | *Tip*

Your calculator might use this key sequence:
5000 $\boxed{\sqrt{\ }}\boxed{=}$ 70.710678
$\boxed{\times}$ .38 $\boxed{=}$ 26.870057

A Checking

1. Estimate each square root to one decimal place using squaring. Show what you did.

a) $\sqrt{15}$ **b)** $\sqrt{30}$

2. Determine each square root to one decimal place using the square root key on your calculator.

a) $\sqrt{8}$ **b)** $\sqrt{42}$ **c)** $\sqrt{163}$ **d)** $\sqrt{979}$

3. Choose one of your answers from question 2 and explain how you know your answer is reasonable.

B Practising

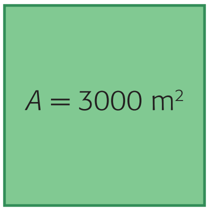
4. Estimate to determine whether each answer is reasonable. Correct any unreasonable answers using the square root key on your calculator.

a) $\sqrt{10} \doteq 3.2$ **c)** $\sqrt{30} \doteq 5.5$ **e)** $\sqrt{342} \doteq 28.5$
b) $\sqrt{15} \doteq 4.8$ **d)** $\sqrt{289} \doteq 27$ **f)** $\sqrt{1482} \doteq 38.5$

5. Calculate each square root to one decimal place. Choose one of your answers and explain why it is reasonable.

a) $\sqrt{18}$ **c)** $\sqrt{38}$ **e)** $\sqrt{800}$

b) $\sqrt{75}$ **d)** $\sqrt{150}$ **f)** $\sqrt{3900}$



6. A square field has an area of 3000 m^2.

a) Explain how you can use $\sqrt{3000}$ to estimate the side length of the square.

b) How do you know the side length is between 50 m and 60 m?

c) Calculate the side length of the square field. Round your answer to one decimal place.

7. What can you add to each number to make a perfect square?

a) 42 **b)** 101 **c)** 399 **d)** 875

8. Tiananmen Square in Beijing, China, is the largest open "square" in any city in the world. It is actually a rectangle of 880 m by 500 m.

a) What would be the approximate side length of a square with the same area as Tiananmen Square?

b) Explain how you know your answer is reasonable.

9. **a)** How do you know the square root of 29 is between 5 and 6?
   b) List three other whole numbers whose square roots are between 5 and 6.

10. Estimate the time an object takes to fall from each height using this formula: time (s) $\doteq 0.45\sqrt{\text{height (m)}}$. Record each answer to one decimal place.

a) 100 m	**c)** 400 m	**e)** 2000 m
b) 200 m	**d)** 900 m	**f)** 10 000 m

11. Kim estimated that the square root of a certain whole number would be close to 5.9. What might the whole number be? Explain your reasoning.

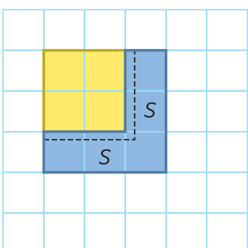
12. **a)** Try Mark's number trick.
    - Choose any whole number greater than 0.
    - Square it.
    - Add twice the original number.
    - Add one.
    - Calculate the square root of the sum.
    - Subtract your original number.
    - Record your answer.

    b) Try Mark's number trick with four other numbers. What do you notice about all your answers?

13. The year 1936 was the last year whose square root was a whole number. What is the next year whose square root will be a whole number? Explain your reasoning.

14. Calculate each square root with a calculator to three decimal places.
    a) $\sqrt{5}$ **b)** $\sqrt{500}$ **c)** $\sqrt{50\,000}$ **d)** $\sqrt{5\,000\,000}$

15. **a)** Describe any patterns you saw in question 14.
    b) Determine $\sqrt{500\,000\,000}$ without a calculator.



16. Explain how to use the diagram to estimate $\sqrt{5}$.

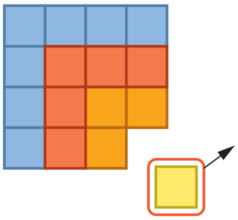
CURIOUS Math

Subtracting to Calculate Square Roots

You can calculate the square root of a perfect square by subtracting consecutive odd numbers, starting with 1. The square root is the number of odd numbers subtracted to get to 0.

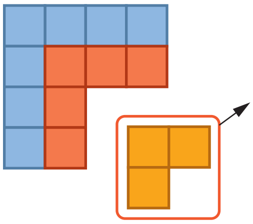
$$\begin{array}{r} 16 \\ \underline{-1} \\ 15 \end{array}$$

one subtraction



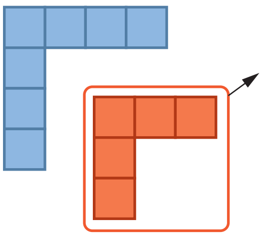
$$\begin{array}{r} 15 \\ \underline{-3} \\ 12 \end{array}$$

two subtractions



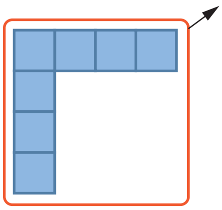
$$\begin{array}{r} 12 \\ \underline{-5} \\ 7 \end{array}$$

three subtractions



$$\begin{array}{r} 7 \\ \underline{-7} \\ 0 \end{array}$$

four subtractions



The first four odd numbers were subtracted from 16 to get 0, so $\sqrt{16} = 4$.

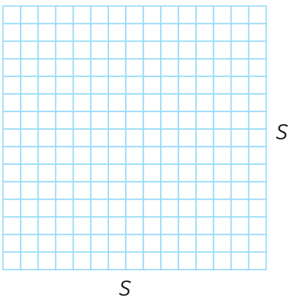
1. Calculate each square root by subtracting consecutive odd numbers, starting at 1.

a) $\sqrt{9}$ **b)** $\sqrt{25}$ **c)** $\sqrt{64}$ **d)** $\sqrt{81}$

Mid-Chapter Review

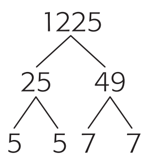
Frequently Asked Questions

Q: How do you determine whether a number is a perfect square?



A1: You can try to draw a square, with whole number side lengths, that has the area of the number. For example, to determine if 225 is a perfect square, try to figure out a whole number side length, s, for a square with that area.

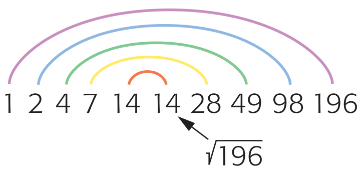
$15^2 = 225$, so $s = 15$, a whole number, and 225 is a perfect square.



A2: You can use prime factors. For example, to determine if 1225 is a perfect square, draw a tree diagram to identify the prime factors. Then group the prime factors to rename 1225 as 35×35 or 35^2. So 1225 is a perfect square.

$$\begin{aligned} 1225 &= (5 \times 7) \times (5 \times 7) \\ &= 35 \times 35 \end{aligned}$$

Q: How do you calculate or estimate a square root?



A1: If a number is a perfect square, you can factor to determine its square root. For example, to calculate $\sqrt{196}$, list all its factors. The partner of 14 is itself, so $14 \times 14 = 14^2$ or 196. $\sqrt{196} = 14$.

A2: If a number is not a perfect square, you have to estimate its square root. For example, to determine $\sqrt{10}$:

Estimate that $\sqrt{10}$ is between 3 and 4 and closer to 3 than 4.	$\sqrt{9} = 3$ $\sqrt{10} = \blacksquare$ $\sqrt{16} = 4$
Square 3.1.	$3.1^2 = 9.61$ (too low)
Square 3.2.	$3.2^2 = 10.24$ (too high)

So $\sqrt{10}$ is between 3.1 and 3.2.

A3: You can use the square root key on a calculator. You can use the square key $\boxed{x^2}$ to check your answer.

Practice

Lesson 1.2

1. Show that each number is a perfect square by drawing a square. Label each side length.

a) 49 **b)** 64 **c)** 144 **d)** 196

2. List the square numbers between 49 and 100. Show your work.

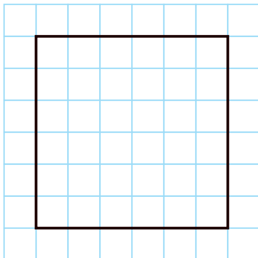
3. Which number is not a perfect square? Show your work.

A. 100 **B.** 121 **C.** 135 **D.** 400

4. Show that 11 025 is a perfect square using its prime factors.
$11\ 025 = 3 \times 3 \times 5 \times 5 \times 7 \times 7$

Lesson 1.3

5. What square number and its square root can be represented by this square? Explain.



6. A square park has an area of 900 m^2. How can you use a square root to determine the side length of the park?

7. How can you use the factors of 81 to determine the square root of 81?

Lesson 1.4

8. Estimate each square root to one decimal place using squaring. Show your work for one answer.

a) $\sqrt{12}$ **b)** $\sqrt{17}$ **c)** $\sqrt{925}$ **d)** $\sqrt{1587}$

9. What is the perimeter of a square with an area of 625 cm^2? Show your work.

1.5 Exploring Problems Involving Squares and Square Roots

YOU WILL NEED

- grid paper
- square tiles
- playing cards (optional)

GOAL

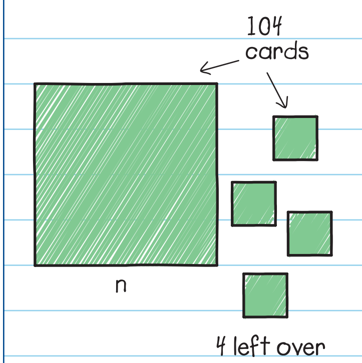
Create and solve problems involving a perfect square.

EXPLORE the Math

Joseph read about a game played with two decks of square playing cards (104 cards). You deal the cards in equal rows and equal columns to form a square. Four cards are left over and not used.

He wanted to know how many rows and columns are in the square.

He drew a diagram and wrote an equation to solve the problem.



$n^2 + 4 = 104$

$100 + 4 = 104$

100 is a square number, so I know I am correct.

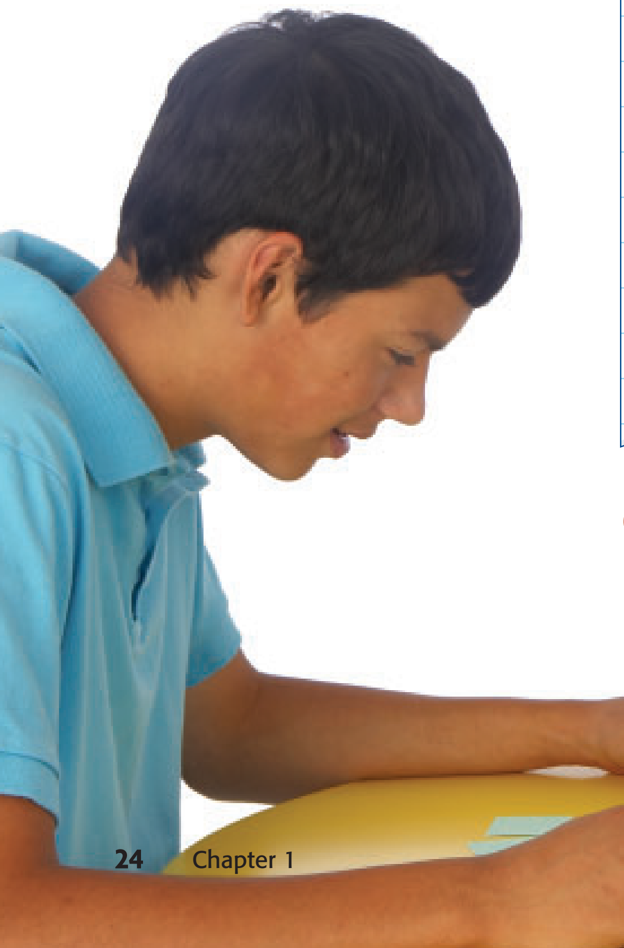
$100 = 10^2$

$n^2 = 10^2$

$n = 10$

The side length of the square is 10, so there are 10 rows and 10 columns of cards.

? What problems can you create that use a square number and another whole number?



Math **GAME**

Tossing Square Roots

YOU WILL NEED
- a die
- a calculator

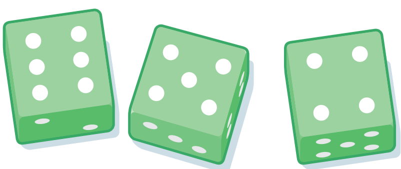
Number of players: 2 to 4

How to Play

1. For each turn, toss a die three times to form a three-digit number.
2. Each player estimates the square root of the tossed number without using a calculator. Each player then records his or her estimate.
3. Each player calculates the square root.
4. Each player scores points for the estimate:
   - Estimate within 2: 1 point
   - Estimate within 1: 2 points
   - Estimate within 0.5: 3 points
5. Continue for five turns. The player who has the most points wins.

Mark's Turn

We rolled 654.



I estimated that the square root of 654 is between 20 and 30 and probably close to 25.

My estimate of 25 is within 1 of the answer. I score 2 points.

1.6 The Pythagorean Theorem

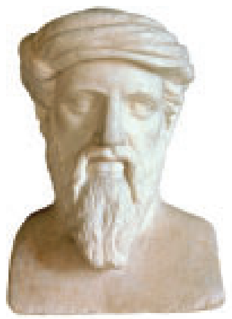
YOU WILL NEED
- grid paper
- a protractor
- a ruler
- a calculator

GOAL

Model, explain, and apply the Pythagorean theorem.

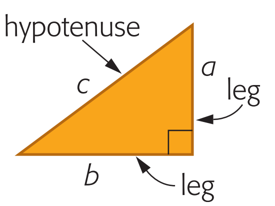
LEARN ABOUT the Math

Guy was doing research on Pythagoras, a mathematician who lived 2500 years ago. Guy discovered that Pythagoras is known for the **Pythagorean theorem,** which is used to solve problems involving the side lengths of **right triangles**. He wondered if this theorem applied to other types of triangles as well.



Pythagorean theorem

a relationship that says the sum of the squares of the lengths of the legs of a right triangle is equal to the square of the length of the hypotenuse. This is written algebraically as $a^2 + b^2 = c^2$.

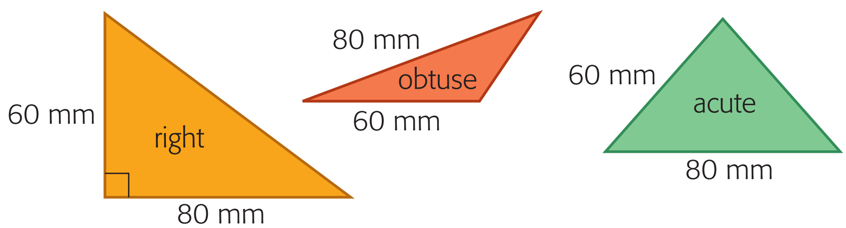


Communication *Tip*

In a right triangle, the two shortest sides are called the legs. The longest side, opposite the right angle, is called the hypotenuse.

Is the Pythagorean theorem true for all types of triangles?

A. Construct two obtuse triangles, two acute triangles, and one right triangle. Each triangle should have one side 60 mm long and another side 80 mm long, such as the ones shown.



B. Measure the third side of each triangle to the nearest millimetre. Record the length of the longest side as $c = \square$ mm. Record the lengths of the two shorter sides as $a = \square$ mm and $b = \square$ mm.

C. For each triangle, calculate $a^2 + b^2$ and c^2. Compare the two values. Record each comparison.

D. Is the Pythagorean theorem true for all types of triangles drawn in your class? Explain.

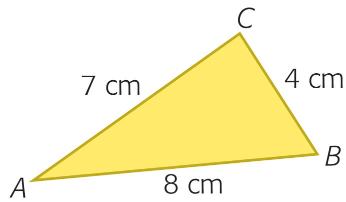
Reflecting

E. Guy drew three triangles, with these results:
Triangle 1: $a^2 + b^2 < c^2$
Triangle 2: $a^2 + b^2 = c^2$
Triangle 3: $a^2 + b^2 > c^2$
What types of triangles did Guy draw? Explain your answer.

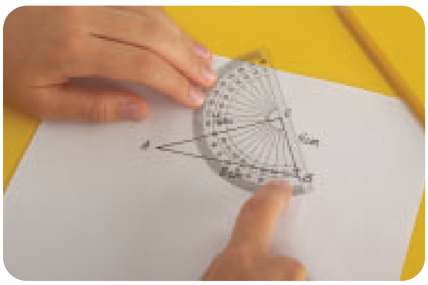
WORK WITH the Math

Example 1 | Identifying a right triangle

Determine whether $\triangle ABC$ is a right triangle.



Elena's Solution



I measured $\angle C$. It is 89°. That is close to 90°, but not exactly 90°, so I am not sure.

$$a^2 + b^2 = 4^2 + 7^2$$
$$= 16 + 49$$
$$= 65$$
$$c^2 = 8^2$$
$$= 64$$

I decided to use the Pythagorean theorem to be sure.

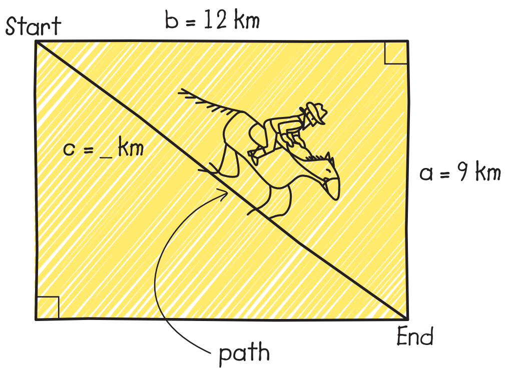
$a^2 + b^2 \neq c^2$
So $\triangle ABC$ is not a right triangle.

$a^2 + b^2$ does not equal c^2.

Example 2 | Using the Pythagorean theorem

A cowhand rode a horse along the diagonal path, instead of around the fence of the ranch. What distance did the cowhand save by riding the diagonal path?

Joseph's Solution



I drew a diagram to represent the problem.

$c^2 = a^2 + b^2$
$c^2 = 9^2 + 12^2$
$= 81 + 144$
$= 225$

I used the Pythagorean theorem to create an equation.
I solved the equation to determine *c*, the length of the hypotenuse.

$c = \sqrt{225}$
$= 15$

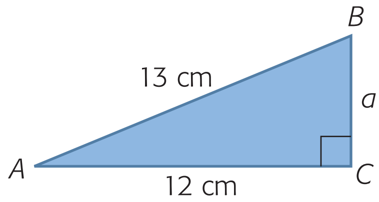
I solved for *c* by calculating the square root.

Distance along fence
= 9 km + 12 km
= 21 km
Distance saved
= 21 km – 15 km
= 6 km.
The cowhand saved 6 km.

I calculated the distance around two sides and the distance the cowhand saved.

Example 3 | Calculating a missing side length

Determine the length of a in $\triangle ABC$.



Vanessa's Solution

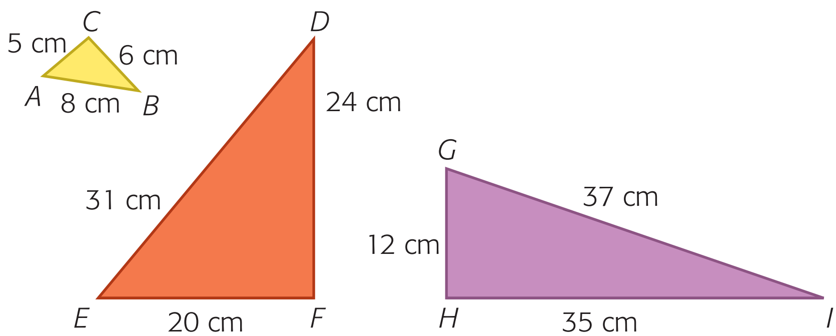
$$a^2 + b^2 = c^2$$
$$a^2 + 12^2 = 13^2$$
$$a^2 + 144 = 169$$
$$a^2 + 144 - 144 = 169 - 144$$
$$a^2 = 25$$
$$a = \sqrt{25}$$
$$= 5$$

The missing length, a, is 5 cm.

$\triangle ABC$ is a right triangle, so I can determine a using the Pythagorean theorem.
I know $b = 12$ and $c = 13$. So I can square these numbers and solve the equation for a.

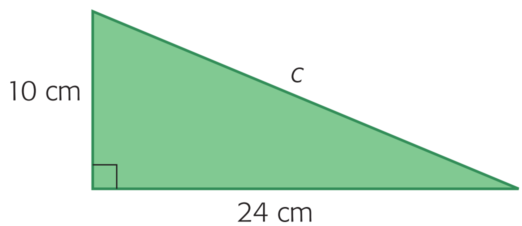
A Checking

1. Which triangle is a right triangle? Show your work.

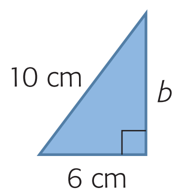


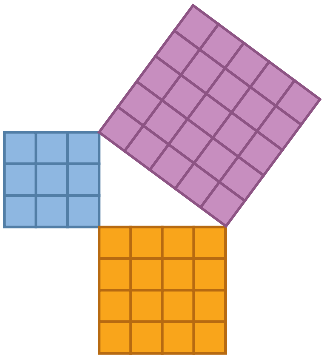
2. Calculate the unknown length in each right triangle. Show your work.

a)



b)





B Practising

3. Hernan formed a triangle with grid paper squares. How can you tell that he formed a right triangle?

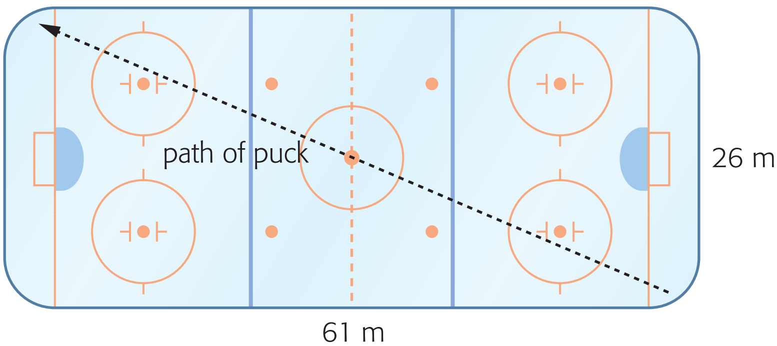
4. **a)** Draw a triangle with side lengths 8 cm, 10 cm, and 13 cm.
b) Does your diagram look like a right triangle? Explain.
c) Show how to use the Pythagorean theorem to determine whether it really is a right triangle.

5. A Pythagorean triple is any set of three whole numbers, a, b, and c, for which $a^2 + b^2 = c^2$. Show that each set of numbers is a Pythagorean triple.

a) 3, 4, 5 **c)** 7, 24, 25 **e)** 9, 40, 41
b) 5, 12, 13 **d)** 8, 15, 17 **f)** 11, 60, 61

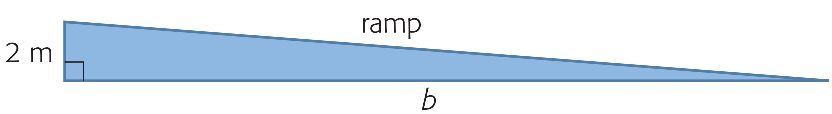
6. **a)** Choose a Pythagorean triple in question 5. Double each number. Is the new triple also a Pythagorean triple? Explain.
b) Choose another Pythagorean triple from question 5. Multiply each number by the same whole number greater than 2. Is the new triple also a Pythagorean triple? Explain.

7. In 2003, the old-time players of the Edmonton Oilers and Montreal Canadiens played an outdoor hockey game before more than 57 000 fans in Commonwealth stadium.



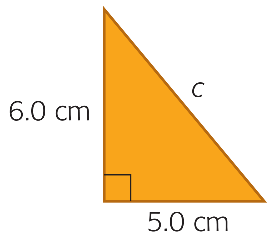
About how far would a hockey puck travel when shot from one corner to the opposite corner?

8. A wheelchair ramp must be 12 m long for every metre of height.
a) What is the length of a ramp that rises 2.0 m?
b) About how long is side b to one decimal place?

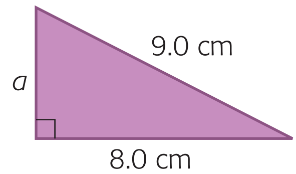


9. Calculate each unknown side to one decimal place.

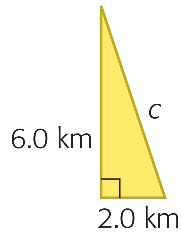
a)



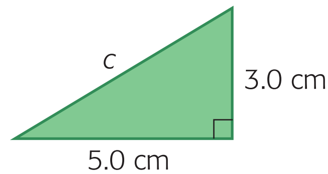
c)



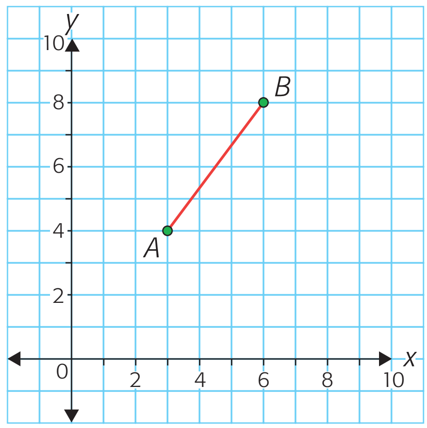
b)



d)

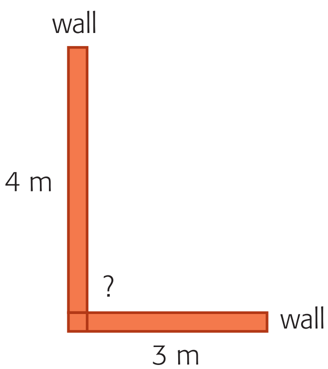


10. What is the distance between points *A* and *B*? Show your work.



11. The hypotenuse of an isosceles right triangle is 10 cm. How long are the legs? Show your work.

12. How can a carpenter use a measuring tape to ensure that the bases of these two walls form a right angle?



13. One side of a right triangle is 9 cm and another side is 12 cm. Draw sketches to show that there are two possible triangles.

14. Why is there only one square but many rectangles with a given diagonal length? Use a diagonal length of 8 cm to help you explain.

1.7 Solve Problems Using Diagrams

YOU WILL NEED

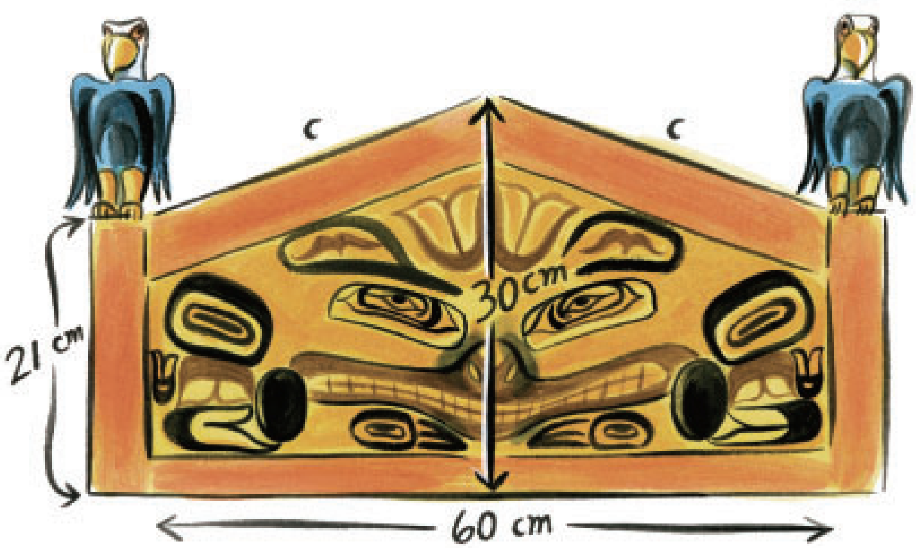
- grid paper
- a calculator
- a ruler

GOAL

Use diagrams to solve problems about squares and square roots.

LEARN ABOUT the Math

Joseph is building a model of the front of a famous Haida longhouse. He wants the model to have these measurements.



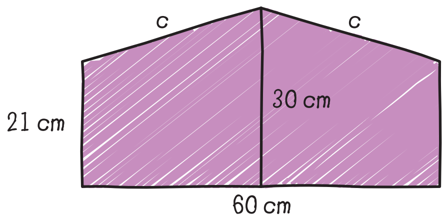
? How can Joseph calculate the two lengths at the top of the model?

Example 1 | Solve a problem by identifying a right triangle

I used a diagram to identify right triangles.

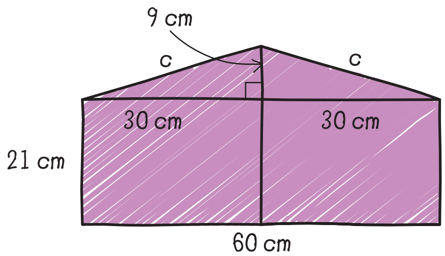
Joseph's Solution

1. Understand the Problem



I drew a diagram that included all I knew about the model. I used c to represent the two lengths I want to know.

2. Make a Plan



Each triangle has a base of half of 60 cm or 30 cm.
The height of each triangle is
30 – 21 = 9 cm.
I can use the Pythagorean theorem to calculate the hypotenuse of each right triangle.

I drew a line to connect the top of the opposite sides of the model. I noticed two right triangles in my diagram.

3. Carry Out the Plan

$$\begin{aligned} c^2 &= 9^2 + 30^2 \\ &= 81 + 900 \\ &= 981 \\ c &= \sqrt{981} \\ &\doteq 31.32 \text{ cm} \end{aligned}$$

Each length at the top of the model is about 31.32 cm.

I know that, in a right triangle, $a^2 + b^2 = c^2$. I used 9 cm for the length a, and 30 cm for the length b.
I solved for c.

Reflecting

A. How did Joseph's diagrams help him solve the problem?

WORK WITH the Math

Example 2 | Visualizing a problem using diagrams

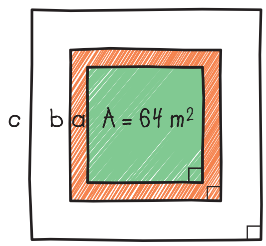
A green square mat in a martial arts competition has an area of 64 m^2. Around the mat is a red danger zone 1 m wide. Around the red zone is a safety area 3 m wide. What is the side length of the overall contest area?

Kaitlyn's Solution

1. Understand the Problem

I have to figure out the overall dimensions of a square mat surrounded by two zones of different widths.

2. Make a Plan



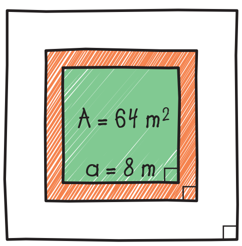
I decided to draw a diagram to help me visualize the mat and two zones. I used letters to show the dimensions that I need to know to figure out the size of the contest area.

3. Carry Out the Plan

$$\text{Area} = a^2$$
$$64 = a^2$$
$$\sqrt{64} = a$$
$$8 \text{ m} = a$$

The square mat is 8 m by 8 m.

First, I calculated the side length of the square mat using the formula for the area of a square.



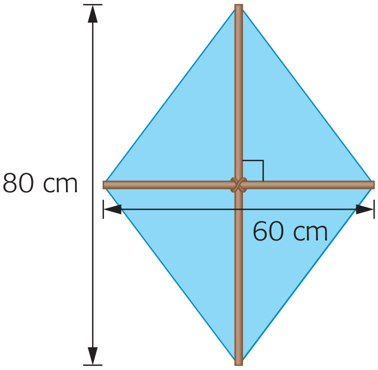
$8 + 3 + 1 + 1 + 3 = 16$ m

The overall contest area is a square measuring 16 m by 16 m.

I added the new information from the calculations to my diagram.

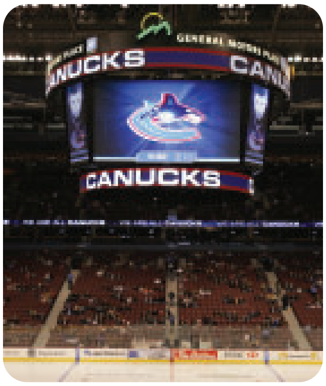
The red zone and the danger zone add 3 m + 1 m to each side of the mat.

A Checking



1. The two cross-pieces of a kite measure 60 cm and 80 cm. The cross-pieces are tied at their middles. What is the perimeter of the kite? Show your work.

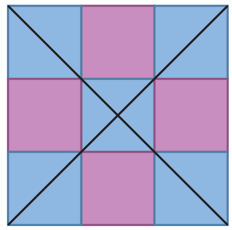
B Practising



2. The LED scoreboard at General Motors Place in Vancouver, BC, has four rectangular video displays. Each display measures about 412 cm by 732 cm. What is the side length of a square with the same area as the four video displays? Show your work.

3. How many squares are on an 8-by-8 chessboard?

4. When Maddy drew a 3-by-3 square, she counted a total of 5 squares along both diagonals.



   a) What is the total number of squares along the two diagonals of a 5-by-5 square? Show your work.

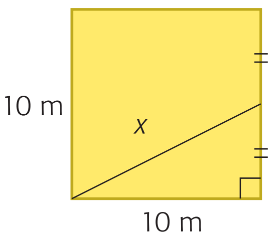
   b) What is the side length of a square with a total of 21 squares along both diagonals? Show your work.

5. The diagonal of a rectangle is 25 cm. The shortest side is 15 cm. What is the length of the other side?

6. Fran cycles 6.0 km north along a straight path. She then rides 10.0 km east. Then she rides 3.0 km south. Then she turns and rides in a straight line back to her starting point. What is the total distance of her ride?

7. The floor of a square room is covered in square tiles. There are 16 tiles on the outside edges of the floor. How many tiles cover the floor?

8. Create and solve a problem about this diagram.



Chapter 1

Chapter Self-Test

1. **a)** What is the least square number greater than 100? Show your work.

b) What is the greatest square number less than 200? Show your work.

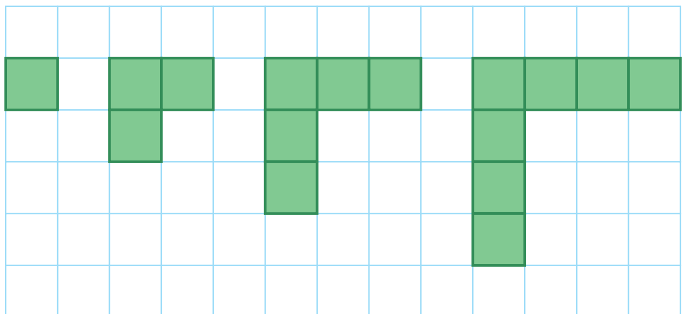
2. **a)** Explain how you know that 25 is a perfect square. Show two different strategies.

b) Express 25 as the sum of two other perfect squares.

3. Each number is the square root of some number. Determine each square number.

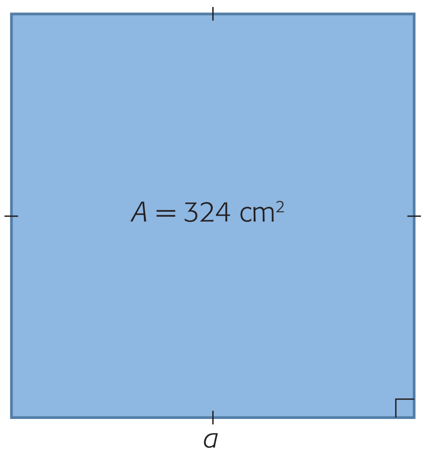
a) 1 **b)** 7 **c)** 15 **d)** 30

4. How many squares can you create by combining one or more of these puzzle pieces? Use linking cubes to help you. Draw each square to show how you arranged the pieces.

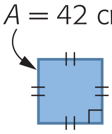


5. Calculate the side length of each square. Show your work.

a)



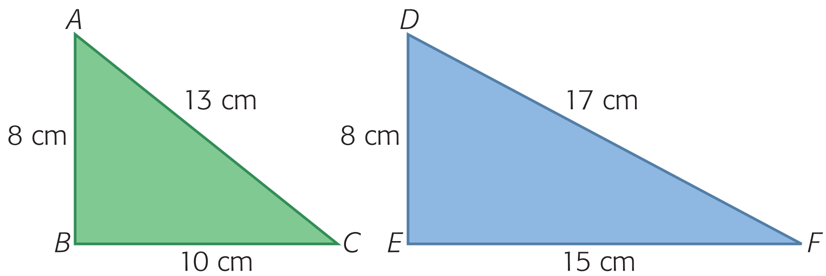
b) $A = 42$ cm^2

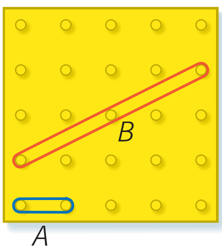


6. Explain how you can estimate $\sqrt{90}$.

7. Saskatchewan is about 652 000 km^2 in area. What would the approximate side lengths be if the province were shaped like a square? Explain.

8. Which of these two triangles is a right triangle? Explain.





9. The length of line segment *A* on the geoboard is 1 unit. What is the length of line segment *B*? Show your work.

10. A square has an area of 100 cm^2. The midpoints of the square are connected to form another square. What are the side lengths of the outer and inner square? Draw a diagram to help you explain.

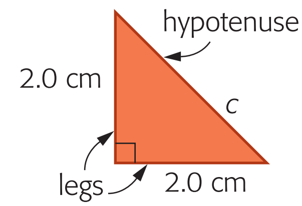
What Do You Think Now?

Revisit What Do You Think? on page 3. How have your answers and explanations changed?

Chapter Review

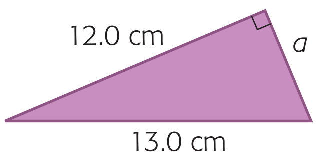
Frequently Asked Questions

Q: How can you use the Pythagorean theorem?



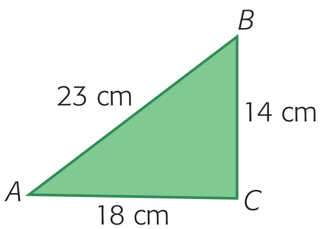
A1: You can calculate the length of the hypotenuse if you know the lengths of the legs. For example, the hypotenuse is about 2.8 cm.

$$\begin{aligned} c^2 &= 2^2 + 2^2 \\ &= 8 \\ c &= \sqrt{8} \\ &\doteq 2.8 \text{ cm} \end{aligned}$$



A2: You can calculate the length of one leg if you know the lengths of the hypotenuse and the other leg. For example, side a is 5 cm.

$$\begin{aligned} a^2 + 12.0^2 &= 13.0^2 \\ a^2 + 144.0^2 &= 169.0 \\ a^2 &= 25.0 \\ a &= \sqrt{25.0} \\ &= 5.0 \text{ cm} \end{aligned}$$



A3: You can determine whether a triangle is a right triangle by comparing $a^2 + b^2$ with c^2.

For example: $$\begin{aligned} a^2 + b^2 &= 14^2 + 18^2 \\ &= 196 + 324 \\ &= 520 \\ c^2 &= 23^2 \\ &= 529 \end{aligned}$$

$520 \neq 529$, so $\triangle ABC$ is not a right triangle.

Practice

Lesson 1.2

1. Determine whether each number is a perfect square using its prime factors. Explain what you did.

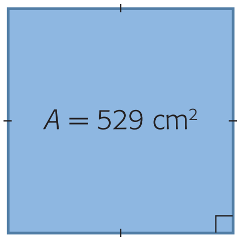
a) $3969 = 3 \times 3 \times 3 \times 3 \times 7 \times 7$

b) $6615 = 3 \times 3 \times 3 \times 7 \times 5 \times 7$

c) $1521 = 3 \times 13 \times 3 \times 13$

d) $125 = 5 \times 5 \times 5$

Lesson 1.3

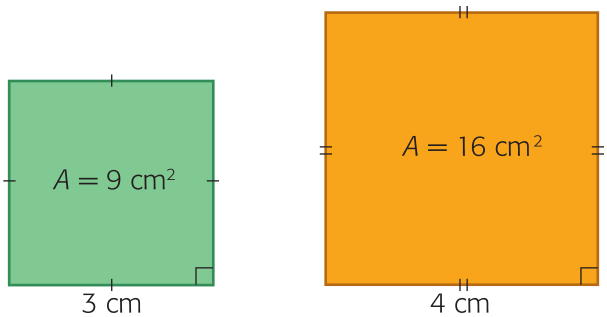


2. Zack drew a square and its area. How can you use his diagram to determine the side length of the square?

3. What is the perimeter of a square parking lot with an area of $1600\ m^2$? Show your work.

Lesson 1.4

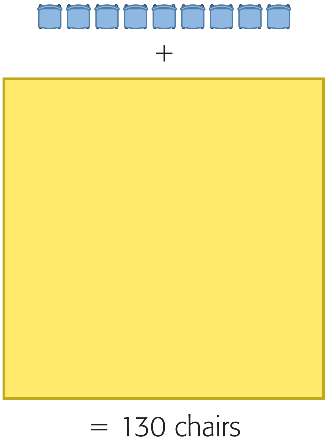
4. How can you use the two squares to show that $\sqrt{11}$ is between 3 and 4?



5. Estimate each square root to one decimal place using squaring. Show your work for one answer.

a) $\sqrt{7}$ **b)** $\sqrt{33}$ **c)** $\sqrt{425}$ **d)** $\sqrt{922}$

6. The official size of a doubles tennis court is 23.9 m by 11.0 m. What is the side length of a square with the same area as a doubles tennis court? Show your work.



Lesson 1.5

7. Chairs in a gym were arranged in the shape of square. Nine chairs were placed in front of the square. A total of 130 chairs were used. How many rows and columns were in the square?

a) Explain how the diagram represents this problem.

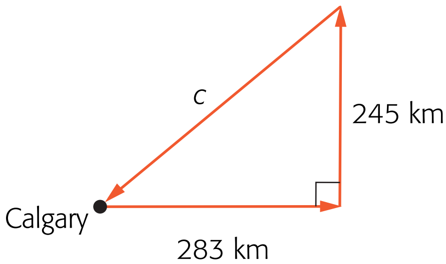
b) What equation would you use to represent this problem?

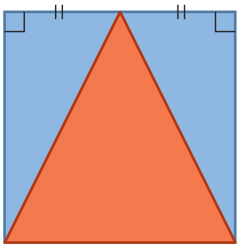
c) Show how to solve the equation.

d) How many rows and columns were in the square?

Lesson 1.6

8. This map shows the route of a helicopter. About how far did the helicopter travel? Show your work.

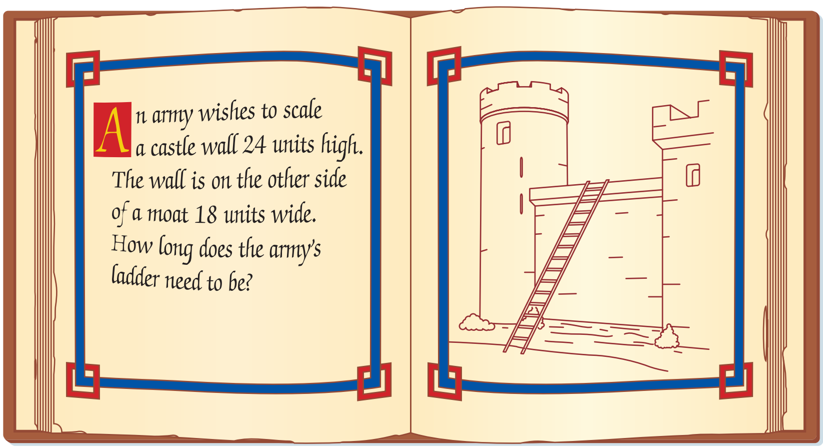




9. The area of the square is 25 cm^2. What are the side lengths of the red triangle?

Lesson 1.7

10. Draw a diagram to solve this problem from a medieval military book. Explain what you did.



Chapter 1

Chapter Task

Task | *Checklist*

✔ Did you estimate to check how reasonable your calculations were?

✔ Did you explain how you chose and solved your equations?

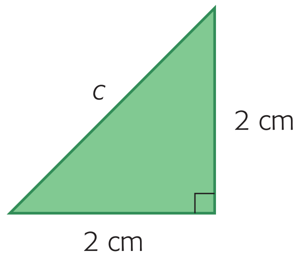
✔ Did you use correct math language?

Pythagorean Spiral

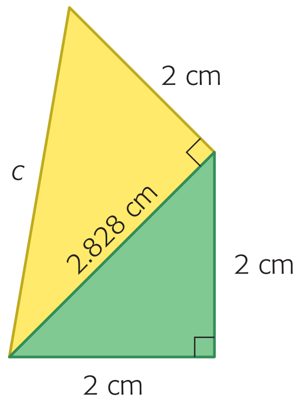
You can use the Pythagorean relationship to create a spiral design.

? How many right triangles do you need to draw to get a hypotenuse just longer than 6 cm?

A. Draw this right triangle in the centre of a large sheet of paper. Use the Pythagorean theorem to show that *c* is about 2.828 cm. How do you know that 2.828 cm is reasonable?

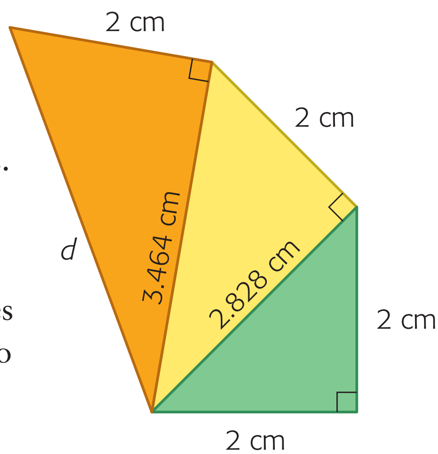


B. Draw a new right triangle on the hypotenuse of the first triangle. Make the outer leg 2 cm long. What is the length of *c*? Round your answer to three decimal places.



C. How do you know your answer in part B is an estimate?

D. Draw another right triangle on the hypotenuse of the second triangle. What is the length of *d*? Round your answer to three decimal places.



E. Repeat drawing right triangles with an outer side of 2 cm long. How many right triangles in total do you need to draw to get a hypotenuse just longer than 6 cm?